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We suggest a “ping-pong” mechanism of enhancement for fusion reactions between a low energy external deuteron beam and the deuterons in a condensed matter or molecular target. The mechanism is based on the possibility of acceleration of a target deuteron by the Coulomb field of a projectile deuteron with its subsequent rebound from a heavy atom in matter and the following fusion of the two deuterons moving towards each other. This effectively converts the fixed target process into a colliding beam reaction. In a simple limiting case this reduces the negative penetrability exponent by a factor of $\sqrt{2}$. We also discuss a contribution given by “zero oscillations” of a bound target deuteron. The proposed mechanism is expected to be efficient in compounds with target deuterons localized in the vicinity of heavy atoms.

It is well known that the fusion at low energies is exponentially suppressed due to the Coulomb repulsion. Consider, for example, two deuterons with a relative velocity v in their center-of-mass reference frame. The Coulomb barrier penetration factor $P(v)$ is given by [1]

$$P(v) \simeq \frac{2\pi e^2}{\hbar v} \exp\left(-\frac{2\pi e^2}{\hbar v}\right) \quad (1)$$

There are a few publications (see, for example, [2–5] and references therein) that claim a substantial enhancement of the DD fusion cross-section in solids. The present paper does not aim to interpret particular experiments. We just analyze some effects in condensed matter or molecules containing deuterons which result in certain enhancement.

Consider a target experiment where one of the deuterons is at rest before the collision whereas another one moves with a velocity v_0 . The total energy of the projectile and target deuterons E_0 at infinity can be decomposed into the energy of relative motion E_r and the energy of the motion of the center-of-mass E_c ,

$$E_0 = \frac{mv_0^2}{2} = E_r + E_c = \frac{\mu v^2}{2} + \frac{Mv_c^2}{2} \quad (2)$$

where m is the deuteron mass, $\mu = m/2$ is the reduced mass, $M = 2m$ is the total mass, v, v_c are the relative velocity and the velocity of the center-of-mass. For a pure two-body problem the relative velocity equals $v = v_0$ and the velocity of the center-of-mass is $v_c = (v_0 + 0)/2 = v_0/2$. Since the barrier penetration factor $P(v)$ is determined by the relative velocity v , half of the energy, the energy of the center-of-mass, is wasted.

Imagine now that as a result of an elastic interaction with, for example, heavy atoms the system transfers part of its momentum \mathbf{q} without changing its energy. This process changes the center-of-mass velocity: $\mathbf{v}_c = \mathbf{v}_0/2 - \mathbf{q}/2m$. Provided that $\mathbf{v}_0 \cdot \mathbf{q} > 0$ the center-of-mass velocity is reduced. According to the energy constrain eq. (2) the relative velocity increases:

$$v_{\mathbf{q}} = [2\mathbf{v}_0^2 - (\mathbf{v}_0 - \mathbf{q}/m)^2]^{1/2}, \quad (3)$$

leading to an exponential enhancement of the penetration factor $P(v)$, see eq. (1). This enhancement can be understood in the following way: as a result of the elastic interaction the two deuterons move towards each other as they do in colliding beams. The maximal enhancement is achieved for $q = mv_0$. Under this condition $v(\mathbf{q})$ is equal to $\sqrt{2}v_0$ instead of v_0 for free motion, i.e. the negative penetrability exponent in eq. (1) is reduced by a factor $\sqrt{2}$. Even a moderate reduction of the center-of-mass momentum can dramatically increase the fusion cross section.

How can the center of mass momentum be reduced? There are two obvious ways to do this:

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- Due to the uncertainty principle the target deuteron is never at rest. The wave function $\phi(\mathbf{q})$ of its original state contains nonzero momentum \mathbf{q} components. Assuming that the target deuteron is confined within a harmonic potential well and thus behaves as a harmonic oscillator with a frequency ω , we have [1]

$$\phi(q) = (\pi\omega\hbar m)^{-1/4} \exp(-q^2/2\omega\hbar m) \quad (4)$$

Here we consider a one dimensional oscillator since only the projection of the target deuteron momentum on the projectile momentum is important.

The barrier penetration factor for fusion can be estimated as

$$P_{\text{eff}} = \int |\phi(q)|^2 P(v_q) dq = P(v_0) \exp\left(\frac{\pi^2 e^4 \omega}{m v_0^4 \hbar}\right), \quad (5)$$

where $P(v_q)$ is given in (1) and the velocity v_q can be obtained expanding (3) as $v_q \approx v_0 + q/m$. Thus, the “zero oscillations” of the target deuteron give an enhancement factor

$$S = \exp\left(\frac{\pi^2 e^4 \omega}{m v_0^4 \hbar}\right) \quad (6)$$

Note that in an excited oscillator state $\langle q^2 \rangle \propto (2n+1)$, where n is the oscillator quantum number. Correspondingly, the enhancement factor S in excited states is much larger.

- Another possibility to modify the center-of-mass momentum arises from the fact that the interaction between the deuterons is a long-range Coulomb interaction which forces the target deuteron to move long before fusion. As a result the target deuteron obtains an additional momentum which it can transfer to the environment. In the simple oscillator model this environment is represented by the harmonic potential well. In other words, the interaction with the projectile deuteron excites the harmonic oscillator. This phenomenon can be understood classically: the projectile deuteron plays ping-pong with the environment using the target deuteron as a ping-pong ball.

Let us present a classical estimate of this effect in a simple model. Assume that both deuterons are influenced by a harmonic oscillator well and interact between themselves by the Coulomb force. To produce fusion the collision of the two deuterons should be characterized by an extremely small impact parameter. Assuming that the harmonic well is isotropic we can neglect the transverse degrees of freedom and reduce the three dimensional problem of the collision of the projectile deuteron with the oscillator to the one-dimensional problem with all particles moving along the x -axis. The Newton equations for the two interacting deuterons are

$$m \frac{d^2 x_t}{dt^2} = -\frac{dU(x_t)}{dx_t} + \frac{e^2}{|x_t - x_p|^2} \quad (7)$$

$$m \frac{d^2 x_p}{dt^2} = -\frac{dU(x_p)}{dx_p} - \frac{e^2}{|x_t - x_p|^2} \quad (8)$$

Here $U(x)$ is the external potential acting on the deuterons. At large distances these equations describe the motion of the projectile with a constant velocity $x_p = v_0 t$ and oscillatory motion of the target near the bottom of the well. Substituting $|x_t - x_p| \approx v_0 t$ in the Coulomb potential energy we find

$$x_t(t) = -\frac{e^2}{v_0^2 m} \int_{-\infty}^{\omega t} \frac{d\tau}{\tau} \cos(\tau - \omega t) \quad (9)$$

$$v_t(t) = \frac{e^2}{v_0^2 m} \left(\frac{1}{|t|} - \omega \int_{-\infty}^{\omega t} \frac{d\tau}{\tau} \sin(\tau - \omega t) \right) \quad (10)$$

The first term in eq.(10) describes the motion in the absence of the oscillator potential and corresponds to the momentum transfer between the deuterons.

We see that at large $|t|$ the velocity of the target deuteron v_t oscillates; each of the oscillations corresponds to a certain momentum transfer to the environment. The total momentum transfer to the environment can be estimated by taking the limit $t = 0$ in the integral in eq.(10):

$$q = \frac{\pi e^2 \omega}{2v_0^2} \quad (11)$$

Substitution of this momentum transfer into eqs.(1,3) gives the enhancement factor S which is exactly equal to the contribution of the zero oscillations, eq.(6). A naive addition of these two effects would give an enhancement factor

$$S = \exp\left(\frac{2\pi^2 e^4 \omega}{mv_0^4 \hbar}\right) \quad (12)$$

(this result is equivalent to the increase of $\langle q^2 \rangle$ by a factor of two; note that in the first excited oscillator state $\langle q^2 \rangle$ is three times larger than that in the ground state.) However, this estimate of the oscillator excitation contribution contains the approximations which can hardly be justified. Indeed, the velocity of the projectile may be taken constant at large distances only. Also, at large distances there is the screening of the deuteron interaction by electrons. Below we present the estimate of the small distance contribution which seems to be more important.

It is convenient to rewrite the deuteron motion equations in terms of the motion of the center-of-mass, $x_c = (x_t + x_p)/2$, and relative motion, $x_r = x_t - x_p$. Taking the sum and the difference of equations (7) and (8) we obtain

$$2m \frac{d^2 x_c}{dt^2} = -\frac{dU(x_t)}{dx_t} - \frac{dU(x_p)}{dx_p} \quad (13)$$

$$\frac{m}{2} \frac{d^2 x_r}{dt^2} = \frac{e^2}{x_r^2} - \frac{1}{2} \left(\frac{dU(x_t)}{dx_t} - \frac{dU(x_p)}{dx_p} \right) \quad (14)$$

If the target deuteron is close to a heavy atom the repulsive force $-\frac{dU(x_t)}{dx_t}$ acting on the target is larger than the similar force $-\frac{dU(x_p)}{dx_p}$ acting on the projectile. This means that the net “external” force acting on the relative motion opposes the Coulomb repulsion and increases the energy of the relative motion! In fact, we have obtained the mechanism to transfer the energy from the motion of the center-of-mass to the relative motion. Note that the harmonic oscillator case is the special one. In this case we have the separation of the variables for the center-of-mass motion and relative motion since $kx_t^2/2 + kx_p^2/2 = kx_c^2 + kx_r^2/4$. The gain in the energy of the relative motion in this case is about half of the deuteron binding energy E_b (in fact, in this model the energy gain is equal to the maximal value of $kx_r^2/4$ at the boundary of the finite size oscillator). This gives the relative velocity $v = \sqrt{2(E_0 + E_b)/m}$ and the enhancement factor

$$S = \exp\left(\frac{2\pi e^2 E_b}{mv_0^3 \hbar}\right) \quad (15)$$

The energy gain can be larger if the deuterons collide in the area of high repulsive potential produced by a heavy atom where the force $-\frac{dU(x_t)}{dx_t} + \frac{dU(x_p)}{dx_p} \sim 2x_r Z_{eff} e^2 / r_c^3$ is large (here r_c is the distance to the heavy nucleus, Z_{eff} is the nuclear charge partly screened by the atomic electrons). Note, however, that at high energy E_0 of the projectile deuteron the displacement of the center-of-mass during the collision is relatively small: $\Delta x_c \sim \frac{r_{min}}{4} (\ln(\frac{4r}{r_{min}}) - 1)$ where $r_{min} = 2e^2/E_0 = 2a_B(27 \text{ eV}/E_0)$ is the classical minimal distance between the deuterons and r is the initial distance between them (it can be identified with the screening radius for the Coulomb interaction between the deuterons), a_B is the Bohr radius. For high energies, Δx_c is small. The deuterons just do not have enough time to reach the area of the high potential U before the fusion. In this case the collision process takes place close to the equilibrium position of the target deuteron and we can approximate the potential U by the oscillator one. Therefore, the estimate eq. (15) seems to be a reasonable approximation for the oscillator excitation effect if the projectile energy is much larger than the atomic unit 27 eV .

To present numerical estimate for the zero oscillation and excitation contributions let us assume that $\hbar\omega = 0.3$ eV and $E_b = 5$ eV. Then it follows from (6),(15) that the enhancement factor is

$$S \sim \exp \left[\left(\frac{300 \text{ eV}}{E_0} \right)^2 + \left(\frac{200 \text{ eV}}{E_0} \right)^{3/2} \right]. \quad (16)$$

This estimate shows that the mechanisms of the exponential enhancement of the fusion described above are efficient only for the low energy deuterons with $E_0 < 1$ KeV. The low-energy enhancement is larger for hard collisions (with momentum transfer $q \sim mv_0$) between the deuterons with rebound from heavy atoms. The hard collisions can transfer all the center-of-mass energy to the relative motion. However, calculation of the hard collision effects is quite cumbersome and it will be published separately [6].

In this paper we present the estimates for a deuteron beam experiment. They can find further applications in non-equilibrium processes like chemical reactions, where cracks in solids or cavities in liquids with an electric field inside can appear. This electric field may accelerate deuterons and create the beam-like situation. Some additional enhancement of the considered effect may be due to the fact that the target deuterons may be in an excited state in a non-equilibrium case. Another possibility may appear in ferroelectric materials with internal electric fields.

The beam-like problem may also appear in a laser-induced fusion where ions are accelerated by the laser field and the interaction with electrons (see e.g. [7]). We propose to add heavy atoms to the deuterium-tritium mixture. The deuteron or tritium rebound from heavy atoms produces a colliding beam and possible enhancement of the fusion.

Note that the density of matter during a laser-induced fusion may be 10^4 times higher than in usual solids (see, e.g. [8]), therefore, the effects of environment are also much larger. For example, the fusion reaction can happen in the area of the strong potential of a heavy atom. As it was discussed above this leads to the energy transfer from the motion of the center-of-mass to the relative motion and exponential enhancement of the fusion probability. The effects of the hard collisions are also very strongly enhanced (up to 10^8 times).

Another important effect of environment - partial screening of the Coulomb barrier by electrons - is discussed, for example, in Ref. [5].

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